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Computers & Operations Research 32 (2005) 1515–1544
computers & operations research
www.elsevier.com/locate/dsw

Travel time analysis of a new automated storage and retrieval system

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Abstract

In conventional automated storage and retrieval systems (AS/RS), stacker cranes are used to access (store or retrieve loads into/from) the storage cells. The stacker cranes can travel simultaneously in the vertical and horizontal directions. However, because the combined motions generally require heavy machineries, the stacker cranes are inadequate for extra heavy loads such as sea container cargo. For such applications, we present a new kind of storage/retrieval (S/R) mechanism, designed with input from AS/RS manufacturers. Unlike stacker cranes, the new S/R mechanism has one vertical platform and N horizontal platforms to serve N tiers of an AS/RS rack. The vertical platform provides the vertical link among different tiers of the AS/RS rack, whereas the horizontal platforms access the storage cells on a given tier. The vertical platform and the horizontal platforms may move independently and concurrently; and the separation of the mechanisms for vertical/horizontal movements also makes the platforms lighter and hence they can operate at a higher speed than the conventional design. We then present a travel-time model under the *stay* dwell point policy, i.e. the platforms remain where they are after completing a storage/retrieval operation. The model is validated by computer simulations. The results show that our analytical model is reliable for the design and analysis of the new kind of AS/RS. We also present guidelines for the optimal design of a rectangular-in-time AS/RS rack with the new S/R mechanism.

Keywords: Automated storage/retrieval system; Travel time model; New storage/retrieval mechanism

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doi:10.1016/j.cor.2003.11.020

1. Introduction

Automated storage/retrieval systems (AS/RS) are storage systems capable of providing random access to all stored items [1]. AS/RS are being widely used in the logistics industry [2]. Major advantages of AS/RS include high throughput, efficient use of space, high reliability and improvement of safety [3]. However, economic factors such as high initial investment, inflexible layout and fixed storage capacity, force us to carefully evaluate the system structure (e.g. the layout and dimensions of the racks, S/R mechanism) and operational policies (e.g. allocation of storage cells and scheduling of the tasks). The objective of our study is to evaluate an AS/RS with a new type of S/R mechanism that has separate vertical and horizontal movement mechanisms.

A conventional AS/RS typically uses stacker crane for reaching and accessing the storage cells. Each stacker crane is equipped with a *vertical drive*, a *horizontal drive* and one or two *shuttle* drives. The vertical drive raises and lowers the load. The horizontal drive moves the load back-and-forth along the aisle. The shuttle drives transfer the loads between the stacker crane's carriages and the storage cells in the AS/RS rack. For greater efficiency, the vertical and horizontal drives are capable of simultaneous operations. The conventional stacker cranes are suitable for certain range of task loads. To handle extra heavy loads (such as loads above 20 tons) at high speed, it is necessary to employ a new S/R mechanism in which the vertical movement and the horizontal movement of the loads are carried out by separate devices, namely, the vertical platform and the horizontal platform. The feasibility of such new design has been verified with AS/RS manufacturers. For convenience, we shall refer to the new types of AS/RS the *split-platform AS/RS*, or *SP-AS/RS* for short. In our project on container handling, we are evaluating the SP-AS/RS for high throughput handling of sea containers.

The main contributions of this paper are:

(i) we present a new S/R mechanism for handling extra heavy loads efficiently;
(ii) we obtain a travel-time model for the new S/R mechanism;
(iii) we provide guidelines for the optimal design of such AS/RS rack.

Section 2 describes the proposed split-platform AS/RS in details, including its structure, operations and advantages. Section 3 reviews existing travel time models for AS/RS. Section 4 presents the travel-time model for the SP-AS/RS. The model is validated by computer simulations in Section 5. In Section 6, guidelines for the optimal design of a rectangular-in-time split-platform AS/RS rack are presented. Finally, promising areas for further study are given in Section 7.

2. A new S/R mechanism of AS/RS for very heavy loads

2.1. Structure and operation of the split-platform AS/RS

As illustrated in Fig. 1, the split-platform AS/RS has one vertical platform (VP) for each rack, and two racks (constituting one aisle) share the horizontal platforms (HPs) on all tiers. The VP provides the vertical link among tiers of the rack, whereas the HPs transfer loads to individual storage cells on the tiers where they are located. The operations of the VP and the HPs are independent. The

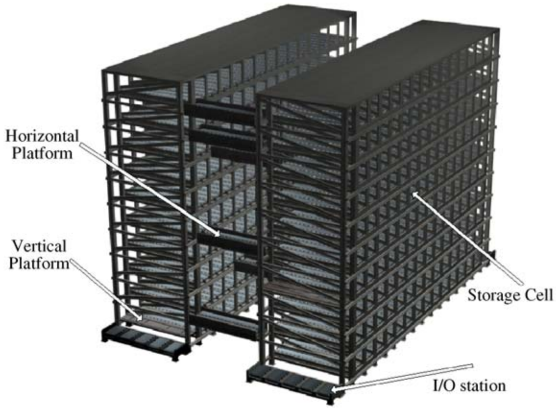

Fig. 1. The new S/R mechanism of AS/RS.

I/O station is located at the ground level on one end of the rack. Two AS/RS manufacturers have confirmed that this new design is both mechanically and economically feasible [21].

The *tiers* (i.e. levels) are numbered by integers from 1 onwards; the *bays* (i.e. columns) are numbered from 0 onwards, all according to their distances from the I/O station. There is no storage cell in bay 0 because it is used by the VP.

The operations of the split-platform AS/RS can be described, respectively, for storage and retrieval operations. During a storage operation:

(1) If not already situated at tier 1, the VP moves from its dwell point to the location. Then the I/O station transfers the load to VP. With the load, the VP goes to the tier assigned to the load. Meanwhile, the HP of the corresponding tier moves from its dwell point to bay 0.
(2) When both are arrived in position, the load is transferred from the VP to the HP.
(3) After the transfer, the VP moves to its dwell point; meanwhile, the corresponding HP carries the load to the destination cell to store the load and then moves to its dwell point if not already in position.

A retrieval operation just reverses the above sequence. Specifically,

(1) The corresponding HP moves from its dwell point to the storage cell to pick up the load, then travels to bay 0; meanwhile, VP moves from its dwell point to the tier assigned to the load.
(2) The load is transferred from the HP to the VP after synchronization.
(3) After the transfer, the VP carries the load to the I/O station at tier 1 and transfers the load to the I/O station, then returns to its dwell point; meanwhile the corresponding HP moves to its dwell point.

2.2. Advantages of the new S/R mechanism

Compared with the traditional stacker crane, the new S/R mechanism offers many advantages. The differences between them are summarized in Table 1 and further explained in the

Table 1
Differences between the SP-AS/RS and the conventional AS/RS

	Conventional AS/RS	SP-AS/RS
Mechanism for handling vertical and horizontal movement of a load	One machine	Separate devices
Lifting capacity	Low	High
Concurrency	Low	High
Performance	Low	High
Rack configuration	Not flexible	Flexible
Fault tolerance	Low	High

following:

- By separating the mechanisms for vertical and horizontal movements, the new AS/RS can handle heavier loads at a higher speed. This has been the most important reason for us to design this SP-AS/RS.
- Because each tier has its own horizontal platform, and all the HPs and VP can run independently and concurrently, the SP-AS/RS can handle many loads at the same time. Obviously this can lead to higher performance.
- Splitting the vertical movement and horizontal movement also simplifies the mechanism of the new AS/RS, and this leads to easier maintenance and reduced downtime of the AS/RS.
- Using the new design, it will be quite convenient to change rack configurations to meet various performance requirements from practice, such as changing the location or number of the VPs and I/O stations.
- With conventional AS/RS, the S/R machine could be the single point of failure. All cells on a rack are affected once it fails. By comparison, with the SP-AS/RS, if an HP is out of order, only the cells within that tier will be affected. Therefore, the new design offers better tolerance of faults.

2.3. Performance comparisons

In this section, the performance of the SP-AS/RS is compared with that of the conventional AS/RS using stacker crane under different rack configurations. Here, we define *throughput* as the reciprocal of the average travel time for the S/R mechanism to handle a job.

The specifications we used are from the following sources of information:

- For stacker crane [22]:
  - Vertical speed: 0.45 m/s,
  - Horizontal speed: 2 m/s;
- For the new S/R mechanism [21]:
  - VP speed: 1 m/s,
  - HP speed: 2 m/s.

Table 2
Performance comparisons between an SP-AS/RS and a conventional one

No. of tiers	No. of bays	S/R mechanism travel time (s)			AS/RS throughput (loads/h)		
		New mechanism	Stacker crane	Improvement (%)	New mechanism	Stacker crane	Improvement (%)
1	288	540.81	540.84	0.01	6.66	6.66	0.00
9	32	78.80	118.44	33.47	45.69	30.40	50.30
12	24	73.02	136.10	46.35	49.30	26.45	86.39
14	21	74.21	151.98	51.17	48.51	23.69	104.80
17	17	78.14	177.19	55.90	46.07	20.32	126.76
24	12	96.90	241.29	59.84	37.15	14.92	149.01
48	6	180.93	477.99	62.15	19.90	7.53	164.19
96	3	358.20	954.97	62.49	10.05	3.77	166.60
288	1	1074.08	2873.90	62.63	3.35	1.25	167.57

To simplify derivations, we assume the pickup and deposit time for loads to be negligible (0) for both cases.

In our experiments, we used a sequence of 100 000 jobs, with equal numbers of storages and retrievals. After each job, the stacker crane stays where it is. For the SP-AS/RS, the VP and the corresponding HP also stay where they are. The travel time shown in Table 2 is the average cycle time for the S/R mechanism and the stacker crane to finish one job.

From Table 2 it is obvious that by using the new S/R mechanism, the performance of AS/RS is substantially improved in terms of travel time and throughput. It also can be seen that the improvement increases with the height of rack. The main reason is that in our experiments the total number of cells within a rack is constant, so the number of bays decreases with the increase of the number of tiers. Given this, the horizontal movement of a load tends to be finished before its vertical movement. This means that a job's handling time is dominated by its vertical movement. By separating the vertical movement from the horizontal movement, the VP of the SP-AS/RS can operate at a higher speed. By comparison, the vertical speed of a stacker crane is relatively slower, so it takes longer time for it to deal with loads on higher tiers.

In brief, the new S/R mechanism we proposed not only can cope with very heavy loads that can not be handled by conventional stacker crane, but it also can offer better performance.

3. Existing travel time models for AS/RS

Many issues related to the efficiency of the AS/RS have been studied in the literature (see references for examples):

(1) The *shape* of the AS/RS. Researchers have built travel-time models for AS/RS racks that are either square-in-time or rectangular-in-time [4,5]. A square-in-time AS/RS is one that takes the same amount of time for the S/R machine to travel to the furthest bay as it does to the highest tier.

(2) The operating characteristics of the S/R machine. In many studies, the S/R machine is assumed to travel at constant speed. Gudehus [6] proposed a method to adjust the previous results when the acceleration and deceleration of the S/R machine are taken into account. Hwang and Lee [7] and Chang et al. [8] developed travel-time models where the speed profiles of the S/R machines in real-world applications were used. Wen et al. [9] studied the impacts of acceleration/deceleration on travel-time models.

(3) The storage cells in an AS/RS rack may be considered homogeneous or they may be partitioned into several areas called *classes*. The partition of cells may be done according to the turnover rate or the dwell time of storage items. Methods for deriving the optimal boundaries for two or three storage regions are proposed by Hausman et al. [4]. Rosenblatt and Eynan [10,11] developed simple recursive procedures to derive optimal boundaries for a general n-class storage rack.

(4) The position of the I/O station(s) is also a factor that affects the AS/RS operation. Bozer and White [5] considered a few alternatives.

(5) The dwell point policy of the S/R machine. This is the policy to decide where the S/R machine will stay when it becomes idle. Bozer and White [5] and Linn and Wysk [12] investigated various dwell point policies. Egbelu [13] developed a linear programming methodology that minimizes the service response time in an AS/RS through the optimal selection of the dwell point of the S/R machine. Park developed two models to obtain optimal dwell point under square-in-time rack with dedicated storage in [14] and uniformly distributed rectangular racks in [15], respectively.

(6) The S/R machine may work in a single-command mode or a dual-command mode. Graves et al. [16], Bozer and White [5] and Pan and Wang [17] studied the two operating modes together with other control policies for AS/RS.

(7) Storage assignment policies. Random assignment, full turnover-based assignment and class-based turnover assignment were investigated by Hausman et al. [4]. Linn and Wysk [12] also considered storage assignment rules with other control decisions.

(8) Request sequencing. Han et al. [18] studied the nearest-neighbor rule for selecting storage locations and sequencing retrieval requests. Seidmann [19] relaxed the assumption that retrievals were selected according to the first-come-first-serve rule when a requested product is stored at multiple locations. van den Berg et al. [20] studied the optimal sequencing of requests with dedicated storage using the block sequencing approach.

4. Travel-time model for the SP-AS/RS

We will analyze the travel time by using continuous models that considerably reduce the difficulty of the subsequent analysis [5]. We will then validate this model by comparing the results predicted by the model with those obtained from computer simulations.

Fig. 2 outlines our approach of derivations.

4.1. Assumptions and notations

The following assumptions are made throughout this paper:

(1) the rack is considered to be a continuous rectangular pick face;
(2) platforms operate on single command basis;

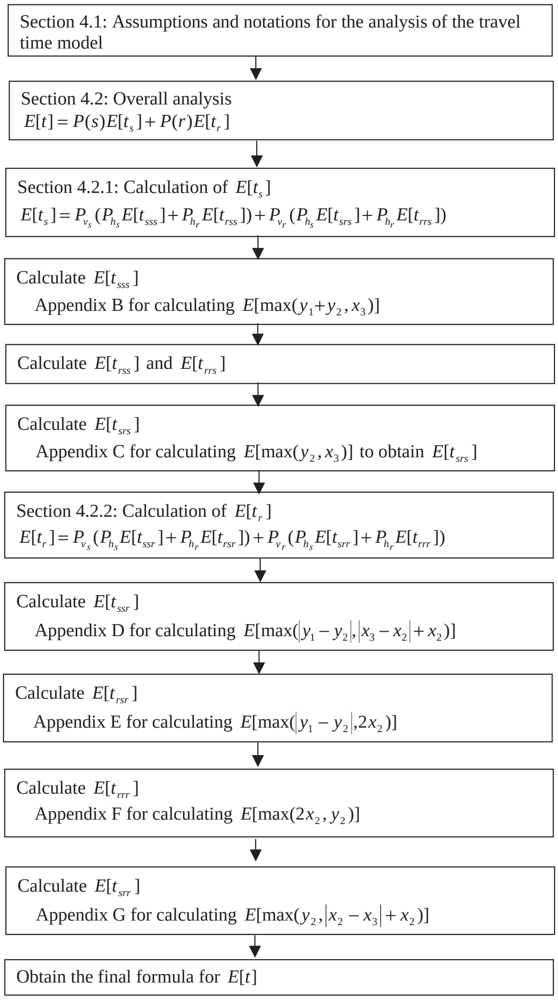

Fig. 2. Organization of Section 4.

(3) unit loads are considered;
(4) randomized storage is used, which means that any point within the pick face is equally likely to be selected for storage or retrieval;
(5) specifications of the rack and the platforms are known. The platforms travel at a constant speed.

We also assume that during each operation, there is no prior information of subsequent jobs, and hence there are no concurrent movements of VP and HPs for different operations.

The following notations are used to standardize the pick face: HL is the length of the rack; VL the height of the rack; hv the speed of the HPs; vv the speed of the VP; t_c the transfer time (this refers to the time needed to transfer a load between VP and HP, or between HP and an AS/RS cell) and t_{c0} the transfer time (this refers to the time needed to transfer a load between the I/O station and the VP).

Let t_{h} denote the travel time required for the HPs to go to the farthest bay from bay 0 and t_{v} the travel time required for the VP to go to the highest tier from tier 1. Then $t_{\mathrm{h}} = HL/hv$ and $t_{\mathrm{v}} = VL/vv$. Let $b = t_{\mathrm{v}}/t_{\mathrm{h}}$. As the value of b may represent the shape of a rack in terms of time, b is referred to as the *shape factor*. With all these symbols, the rack can be normalized as a rectangular pick face with length of 1 and height of b in terms of time.

At the same time, let $c = t_c/t_{\mathrm{h}}$ and $c_0 = t_{c_0}/t_{\mathrm{h}}$ in order to normalize the transfer times.

With stacker cranes, the symmetry of the vertical and horizontal movements allows to assume that $0 < b \leqslant 1$. With the new S/R mechanism, we will let b to be an arbitrary positive value.

In this model, the dwell point policy is the '*stay policy*' which means the platforms stay where they are after the completion of each storage or retrieval operation. The travel time model under the dwell point policy that after each operation the VP goes back to tier 1 and the corresponding HP returns to bay 0 is described in [23].

4.2. Analysis

Assume that the ratio for storage operations is α in an arbitrary finite job sequence, and let (x_1, y_1) and (x_2, y_2) denote the previous operation point and the target point of current job, respectively. Let (x_3, y_2) denote the last operation point whose y coordinate is also y_2. All these coordinates' values are in terms of time.

As randomized storage is used, the probability distribution function and probability density function of y_i $(i = 1, 2)$ are as follows:

$$F_{y_i}(v) = \begin{cases} \dfrac{v}{b}, & 0 \leqslant v \leqslant b, \\ 1, & v \geqslant b \end{cases} \quad \text{and} \quad f_{y_i}(v) = \begin{cases} \dfrac{1}{b}, & 0 \leqslant v \leqslant b, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Thus

$$E[y_i] = \int_0^b v \times f_{y_i}(v)\,\mathrm{d}v = \frac{b}{2}.$$

Meanwhile, the probability distribution function and the probability density function of x_i $(i = 1, 2, 3)$ are

$$F_{x_i}(v) = \begin{cases} v, & 0 \leqslant v \leqslant 1, \\ 1, & v \geqslant 1 \end{cases} \quad \text{and} \quad f_{x_i}(v) = \begin{cases} 1, & 0 \leqslant v \leqslant 1, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Hence

$$E[x_i] = \int_0^1 v \times f_{x_i}(v)\,\mathrm{d}v = \frac{1}{2}.$$

As different expressions must be used to obtain the expected travel time for a storage operation and a retrieval operation, it is necessary to distinguish the operation type in order to obtain the formula to describe the expected travel time. The formula is

$$E[t] = P(s)E[t_s] + P(r)E[t_r]$$

where t denotes the cycle time for the S/R mechanism to complete an operation. t_s indicates the time spent if the current job is storage, while t_r is the time spent for a retrieval operation. Thus, it is clear that $E[t]$ denotes the expected travel time for one operation, $E[t_s]$ gives the expected travel time if the current job is storage and $P(s)$ is the probability for the current job to be storage. $E[t_r]$ and $P(r)$ are similarly defined for the case of retrieval. By definition, $P(r) = 1 - P(s)$.

Now we calculate $E[t_s]$ and $E[t_r]$ one by one.

4.2.1. The calculation of $E[t_s]$

The expected travel time to point (x_2, y_2) is related to the operation type of the previous operation and the last operation at tier y_2. The reason is that if an operation is retrieval, the VP always returns to tier 1 and the corresponding HP goes back to bay 0 according to the dwell point policy. Whereas if the operation is storage, the dwell points of both the VP and the HP are arbitrary and comply with uniform distribution since randomized storage assignment policy is assumed. Position of the VP is decided by the previous operation, and the last operation at tier y_2 decides the location of the corresponding HP.

Considering all the permutations of the previous operation and the last operation at tier y_2, the expression of the expected travel time of the storage cycle is

$$E[t_s] = P_{v_s}E[t_{ss}] + P_{v_r}E[t_{rs}] = P_{v_s}(P_{h_s}E[t_{sss}] + P_{h_r}E[t_{rss}]) + P_{v_r}(P_{h_s}E[t_{srs}] + P_{h_r}E[t_{rrs}]). \quad (3)$$

The meanings of the subscripts in Eq. (3) are as follows: For the format of '*ab*', the former letter denotes the type of the previous operation and the latter one denotes that of the current operation. For example, '*rs*' represents that the current operation to be done is storage, and the operation before it is retrieval. For the format of '*abc*', the first letter denotes the type of the last operation at tier y_2, the middle one denotes the type of the previous operation and the last one denotes the type of the current operation. For instance, '*rsr*' represents that the current operation to be done is retrieval, the operation before it is storage, and the last operation with the operation point which has the same y coordinate as that of the current job is also retrieval.

$E[t_{ab}]$ or $E[t_{abc}]$ represents the expected travel time of the current operation. P_{v_s} is the probability that the last operation of the VP is storage. P_{v_r} is the probability that the last operation of the VP is retrieval. P_{h_s} is the probability that the last operation of the HP related with the current operation is storage. P_{h_r} is the probability that the last operation of the HP related with the current operation is retrieval.

Having assumed that randomized storage policy is used, we have

$$P_{v_s} = \alpha, \quad P_{v_r} = 1 - P_{v_s} = 1 - \alpha,$$

$$P_{h_s} = \alpha, \quad P_{h_r} = 1 - P_{h_s} = 1 - \alpha.$$

(1) The calculation of $E[t_{sss}]$. In this case, the current job is storage; the previous operation and the last operation at tier y_2 are also storage operations. Then the formula for the travel time here is (the derivation is shown in Appendix A)

$$t_{sss} = \max(y_1 + c_0 + y_2, x_3) + x_2 + 2c.$$

As the transfer time t_{c_0} and t_c are rather small compared with the time spent on platform movements, and also in order to simplify the following derivations, we assume t_{c_0} and t_c to be 0 henceforth. This implies that $c = 0$ and $c_0 = 0$.

So in this case,

$$t_{sss} = \max(y_1 + y_2, x_3) + x_2.$$

It is easy to see that

$$E[t_{sss}] = E[x_2] + E[\max(y_1 + y_2, x_3)] = E[x_2] + E[Z],$$

where $Z = \max(y_1 + y_2, x_3)$. From Appendix B, we have

$$E[Z] = \begin{cases} \frac{7}{12}b^2 + \frac{1}{2}, & 0 < b \leqslant \frac{1}{2}, \\ -\frac{1}{12}b^2 + \frac{4}{3}b + \frac{1}{3b} - \frac{1}{24b^2} - \frac{1}{2}, & \frac{1}{2} \leqslant b \leqslant 1, \\ b + \frac{1}{24b^2}, & b \geqslant 1. \end{cases}$$

Meanwhile, as $E[x_2] = \frac{1}{2}$, the expected travel time is

$$E[t_{sss}] = \begin{cases} \frac{7}{12}b^2 + 1, & 0 < b \leqslant \frac{1}{2}, \\ -\frac{1}{12}b^2 + \frac{4}{3}b + \frac{1}{3b} - \frac{1}{24b^2}, & \frac{1}{2} \leqslant b \leqslant 1, \\ b + \frac{1}{24b^2} + \frac{1}{2}, & b \geqslant 1. \end{cases} \quad (4)$$

(2) The calculation of $E[t_{rss}]$. In this case, the current job is storage, the previous operation is also storage and the last operation at tier y_2 is retrieval. Thus the formula for the travel time in this case is given by

$$t_{rss} = y_1 + y_2 + x_2 + 2c + c_0.$$

According to the assumption $c = 0$ and $c_0 = 0$, we have,

$$t_{rss} = y_1 + y_2 + x_2.$$

Thus

$$E[t_{rss}] = E[y_1] + E[y_2] + E[x_2] = \tfrac{1}{2}b + \tfrac{1}{2}b + \tfrac{1}{2} = b + \tfrac{1}{2}. \quad (5)$$

(3) The calculation of $E[t_{rrs}]$. In this case, the current job is storage. The previous operation is retrieval and the last operation at tier y_2 is also retrieval. The formula for the travel time in this case is

$$t_{rrs} = y_2 + x_2 + 2c + c_0.$$

As we assume that $c = 0$ and $c_0 = 0$, we have

$$t_{rrs} = y_2 + x_2.$$

Thus

$$E[t_{rss}] = E[y_2] + E[x_2] = \tfrac{1}{2}\, b + \tfrac{1}{2}. \tag{6}$$

(4) The calculation of $E[t_{srs}]$. In this case, the current job is storage. The previous operation is retrieval and the last operation at tier y_2 is storage. The formula for the travel time in this case is

$$t_{srs} = \max(y_2 + c_0, x_3) + x_2 + 2c.$$

As assumed, $c = 0$ and $c_0 = 0$, it has

$$t_{srs} = \max(y_2, x_3) + x_2.$$

Let Z denote $\max(y_2, x_3)$. From Appendix C, we have

$$E[Z] = \begin{cases} \dfrac{1}{6}\, b^2 + \dfrac{1}{2}, & 0 < b \leqslant 1, \\[2ex] \dfrac{b}{2} + \dfrac{1}{6b}, & b \geqslant 1. \end{cases}$$

Hence,

$$E[t_{srs}] = E[Z] + E[x_2] = \begin{cases} \dfrac{1}{6}\, b^2 + 1, & 0 < b \leqslant 1, \\[2ex] \dfrac{b}{2} + \dfrac{1}{6b} + \dfrac{1}{2}, & b \geqslant 1. \end{cases} \tag{7}$$

Recall that

$$\begin{aligned} E[t_s] &= P_{\mathrm{v}_s}(P_{\mathrm{h}_s} E[t_{sss}] + P_{\mathrm{h}_r} E[t_{rss}]) + P_{\mathrm{v}_r}(P_{\mathrm{h}_s} E[t_{srs}] + P_{\mathrm{h}_r} E[t_{rrs}]) \\ &= \alpha^2 E[t_{sss}] + \alpha(1-\alpha)(E[t_{rss}] + E[t_{srs}]) + (1-\alpha)^2 E[t_{rrs}]. \end{aligned} \tag{8}$$

Substituting Eqs. (4)–(7) into Eq. (8) yields

$$E[t_s] = \begin{cases} \left(\dfrac{5}{12}\,\alpha^2 + \dfrac{1}{6}\,\alpha\right) b^2 + \left(\dfrac{1}{2} - \dfrac{1}{2}\,\alpha^2\right) b + \left(\dfrac{1}{2}\,\alpha + \dfrac{1}{2}\right), & 0 < b \leqslant \dfrac{1}{2}, \\[2ex] \dfrac{-3\alpha^2 + 2\alpha}{12}\, b^2 + \left(\dfrac{5}{6}\,\alpha^2 + \dfrac{1}{2}\right) b + \dfrac{\alpha^2}{3}\,\dfrac{1}{b} - \dfrac{\alpha^2}{24}\,\dfrac{1}{b^2} & \\ \quad + \dfrac{1}{2}\,(\alpha - 2\alpha^2 + 1), & \dfrac{1}{2} \leqslant b \leqslant 1, \\[2ex] \dfrac{\alpha + 1}{2}\, b + \dfrac{\alpha - \alpha^2}{6b} + \dfrac{\alpha^2}{24b^2} + \dfrac{1}{2}, & b \geqslant 1. \end{cases} \tag{9}$$

4.2.2. The calculation of $E[t_r]$

The expected travel time of a retrieval cycle is

$$E[t_r] = P_{v_s}E[t_{sr}] + P_{v_r}E[t_{rr}]$$
$$= P_{v_s}(P_{h_s}E[t_{ssr}] + P_{h_r}E[t_{rsr}]) + P_{v_r}(P_{h_s}E[t_{srr}] + P_{h_r}E[t_{rrr}]),$$

where the meanings of the symbols are the same as those in Eq. (3). We will give our calculation in the following steps.

(1) The calculation of $E[t_{ssr}]$. In this case, the current job is retrieval. The previous operation and the last operation at tier y_2 are all storages. The formula for the travel time in this case is

$$t_{ssr} = \max(|y_1 - y_2|, |x_3 - x_2| + x_2 + c) + y_2 + c + c_0.$$

Since we assume that $c = 0$ and $c_0 = 0$, it has

$$t_{ssr} = \max(|y_1 - y_2|, |x_3 - x_2| + x_2) + y_2.$$

Hence $E[t_{ssr}] = E[\max(|y_1 - y_2|, |x_3 - x_2| + x_2)] + E[y_2]$.

Let Z denote the expression $\max(|y_1 - y_2|, |x_3 - x_2| + x_2)$. From the results of Appendix D, we have

$$E[Z] = \begin{cases} \frac{1}{40}b^3 + \frac{5}{6}, & 0 < b \leqslant 1, \\ -\frac{b^3}{120} + \frac{b^2}{12} + \frac{1}{6b} - \frac{1}{20b^2} + \frac{2}{3}, & 1 \leqslant b \leqslant 2, \\ \frac{b}{3} + \frac{5}{6b} - \frac{19}{60b^2}, & b \geqslant 2. \end{cases}$$

Hence,

$$E[t_{ssr}] = E[Z] + E[y_2] = \begin{cases} \frac{1}{40}b^3 + \frac{b}{2} + \frac{5}{6}, & 0 < b \leqslant 1, \\ -\frac{b^3}{120} + \frac{b^2}{12} + \frac{b}{2} + \frac{1}{6b} - \frac{1}{20b^2} + \frac{2}{3}, & 1 \leqslant b \leqslant 2, \\ \frac{5b}{6} + \frac{5}{6b} - \frac{19}{60b^2}, & b \geqslant 2. \end{cases} \quad (10)$$

(2) The calculation of $E[t_{rsr}]$. In this case, the current job is retrieval. The previous operation is storage and the last operation at tier y_2 is retrieval. Then the formula for the calculation of the travel time here is

$$t_{rsr} = \max(|y_1 - y_2|, 2x_2 + c) + y_2 + c + c_0.$$

We have assumed that $c = 0$ and $c_0 = 0$, so it has,

$$t_{rsr} = \max(|y_1 - y_2|, 2x_2) + y_2.$$

So the expected retrieval time will be

$$E[t_{rsr}] = E[\max(|y_1 - y_2|, 2x_2)] + E[y_2].$$

Let Z denote $\max(|y_1 - y_2|, 2x_2)$, and from Appendix E, we have

$$E[Z] = \begin{cases} \dfrac{b^2}{24} + 1, & 0 < b \leqslant 2, \\ \dfrac{b}{3} + \dfrac{4}{3b} - \dfrac{2}{3b^2}, & b \geqslant 2. \end{cases}$$

Hence,

$$E[t_{rsr}] = E[Z] + E[y_2] = \begin{cases} \dfrac{b^2}{24} + \dfrac{b}{2} + 1, & 0 < b \leqslant 2, \\ \dfrac{5b}{6} + \dfrac{4}{3b} - \dfrac{2}{3b^2}, & b \geqslant 2. \end{cases} \quad (11)$$

(3) The calculation of $E(t_{rrr})$. In this case, the current job is retrieval. The previous operation and the last operation at tier y_2 are both retrievals. Thus, the formula for the travel time in this case is

$$t_{rrr} = \max(y_2, 2x_2 + c) + y_2 + c + c_0.$$

According to our assumption, $c = 0$ and $c_0 = 0$, we have

$$t_{rrr} = \max(y_2, 2x_2) + y_2.$$

Hence,

$$E[t_{rrr}] = E[\max(2x_2, y_2)] + E[y_2] = E[Z] + E[y_2],$$

where $Z = \max(2x_2, y_2)$.

From Appendix F, we have

$$E[Z] = \begin{cases} \dfrac{b^2}{12} + 1, & 0 < b \leqslant 2, \\ \dfrac{b}{2} + \dfrac{2}{3b}, & b \geqslant 2. \end{cases}$$

Therefore,

$$E[t_{rrr}] = E[y_2] + E[Z] = \begin{cases} \dfrac{b^2}{12} + \dfrac{b}{2} + 1, & 0 < b \leqslant 2, \\ b + \dfrac{2}{3b}, & b \geqslant 2. \end{cases} \quad (12)$$

(4) The calculation of $E[t_{srr}]$. In this case, the current job is retrieval. The previous operation is also retrieval and the last operation at tier y_2 is storage. The formula for the travel time in this case is

$$t_{srr} = \max(y_2, |x_2 - x_3| + x_2 + c) + y_2 + c + c_0.$$

Due to the assumption, $c = 0$ and $c_0 = 0$, it has

$$t_{srr} = \max(y_2, |x_2 - x_3| + x_2) + y_2.$$

Let Z denote $\max(y_2, |x_2 - x_3| + x_2)$, and we have

$$E[t_{srr}] = E[Z] + E[y_2].$$

From Appendix G, we have

$$E[Z] = \begin{cases} \dfrac{1}{16}b^3 + \dfrac{5}{6}, & 0 < b \leqslant 1, \\ -\dfrac{1}{48}b^3 + \dfrac{1}{6}b^2 + \dfrac{1}{12b} + \dfrac{2}{3}, & 1 \leqslant b \leqslant 2, \\ \dfrac{b}{2} + \dfrac{5}{12b}, & b \geqslant 2. \end{cases}$$

Therefore,

$$E[t_{srr}] = E[Z] + E[y_2] = \begin{cases} \dfrac{1}{16}b^3 + \dfrac{1}{2}b + \dfrac{5}{6}, & 0 < b \leqslant 1, \\ -\dfrac{1}{48}b^3 + \dfrac{1}{6}b^2 + \dfrac{1}{2}b + \dfrac{1}{12b} + \dfrac{2}{3}, & 1 \leqslant b \leqslant 2, \\ b + \dfrac{5}{12b}, & b \geqslant 2. \end{cases} \tag{13}$$

Finally, with Eqs. (10)–(13), we can have the expected travel time of a retrieval operation. Recall that

$$\begin{aligned} E[t_r] &= P_{v_s}E[t_{sr}] + P_{v_r}E[t_{rr}] \\ &= P_{v_s}(P_{h_s}E[t_{ssr}] + P_{h_r}E[t_{rsr}]) + P_{v_r}(P_{h_s}E[t_{srr}] + P_{h_r}E[t_{rrr}]) \\ &= \alpha(\alpha E[t_{ssr}] + (1-\alpha)E[t_{rsr}]) + (1-\alpha)(\alpha E[t_{srr}] + (1-\alpha)E[t_{rrr}]). \end{aligned} \tag{14}$$

So,

$$E[t_r] = \begin{cases} \dfrac{-3\alpha^2 + 5\alpha}{80}b^3 + \dfrac{\alpha^2 - 3\alpha + 2}{24}b^2 + \dfrac{1}{2}b + \dfrac{6-\alpha}{6}, & 0 < b \leqslant 1, \\ \dfrac{3\alpha^2 - 5\alpha}{240}b^3 + \dfrac{-\alpha^2 + \alpha + 2}{24}b^2 + \dfrac{1}{2}b + \dfrac{\alpha^2 + \alpha}{12b} & \\ \quad -\dfrac{\alpha^2}{20b^2} + \dfrac{-\alpha + 3}{3}, & 1 \leqslant b \leqslant 2, \\ \dfrac{-\alpha + 6}{6}b + \dfrac{-3\alpha^2 + 5\alpha + 8}{12b} + \dfrac{21\alpha^2 - 40\alpha}{60b^2}, & b \geqslant 2. \end{cases} \tag{15}$$

Now, we are ready for the expected travel time $E[t]$. Recall that $E[t]=P(s)E[t_s]+P(r)E[t_r]$. And obviously, $P(s)=\alpha$ and $P(r)=1-\alpha$. Then based on Eqs. (9) and (15), we have

$$E[t]=\begin{cases} \dfrac{3\alpha^3-8\alpha^2+5\alpha}{80}b^3+\dfrac{9\alpha^3+8\alpha^2-5\alpha+2}{24}b^2 \\ \quad +\dfrac{1-\alpha^3}{2}b+\dfrac{3-2\alpha+2\alpha^2}{3}, & 0<b\leqslant\dfrac{1}{2}, \\ \dfrac{3\alpha^3-8\alpha^2+5\alpha}{80}b^3+\dfrac{-7\alpha^3+8\alpha^2-5\alpha+2}{24}b^2 \\ \quad +\dfrac{3+5\alpha^3}{6}b+\dfrac{\alpha^3}{3b}-\dfrac{\alpha^3}{24b^2}+\dfrac{3-2\alpha+2\alpha^2-3\alpha^3}{3}, & \dfrac{1}{2}\leqslant b\leqslant 1, \\ \dfrac{-3\alpha^3+8\alpha^2-5\alpha}{240}b^3+\dfrac{\alpha^3-2\alpha^2-\alpha+2}{24}b^2 \\ \quad +\dfrac{\alpha^2+1}{2}b+\dfrac{-3\alpha^3+2\alpha^2+\alpha}{12b}+\dfrac{11\alpha^3-6\alpha^2}{120b^2}+\dfrac{2\alpha^2-5\alpha+6}{6}, & 1\leqslant b\leqslant 2, \\ \dfrac{2\alpha^2-2\alpha+3}{3}b+\dfrac{\alpha^3-6\alpha^2-3\alpha+8}{12b}+\dfrac{-37\alpha^3+122\alpha^2-80\alpha}{120b^2} \\ \quad +\dfrac{\alpha}{2}, & b\geqslant 2. \end{cases} \tag{16}$$

In the case of infinite sequence of jobs, the value of α will be $\frac{1}{2}$. This special case of Eq. (16) can be expressed as

$$E[t]_{\alpha=\frac{1}{2}}=\begin{cases} \dfrac{7}{160}b^3+\dfrac{7}{64}b^2+\dfrac{7}{16}b+\dfrac{5}{6}, & 0<b\leqslant\dfrac{1}{2}, \\ \dfrac{7}{160}b^3+\dfrac{5}{192}b^2+\dfrac{29}{48}b+\dfrac{1}{24b}-\dfrac{1}{192b^2}+\dfrac{17}{24}, & \dfrac{1}{2}\leqslant b\leqslant 1, \\ -\dfrac{7}{1920}b^3+\dfrac{3}{64}b^2+\dfrac{5}{8}b+\dfrac{5}{96b}-\dfrac{1}{960b^2}+\dfrac{2}{3}, & 1\leqslant b\leqslant 2, \\ \dfrac{5}{6}b+\dfrac{41}{96b}-\dfrac{113}{960b^2}+1, & b\geqslant 2. \end{cases}$$

5. Validation of the travel-time model

To evaluate the continuous model for its accuracy, we compare the results obtained from the model with those from the computer simulations. The algorithm for the simulations is shown in Fig. 3.

Two sets of configurations are used in the simulations.

Configuration 1 [21]:

(a) the number of total cells in the rack is 288;

```
Generalize the job sequence with the percentage of storage operations being α ;
Initialize the rack;

While ( the job sequence has not been finished )
    Randomly generate a position in the rack for the current job;
    If ( the current job is a storage operation )
        If ( the position for the current operation  is empty  )
            Execute the storage operation;
        Else if ( there are any empty cells in the rack )
            Find an empty position randomly and execute the storage operation;
          Else terminate this simulation;
    Else  // the current job is a retrieval operation
        If  ( the position for the current operation is full )
            Execute the retrieval operation;
        Else if ( there are any occupied cells in the rack )
            Find an occupied position randomly and execute the retrieval operation;
          Else terminate this simulation;
Calculate the average travel time of the S/R mechanism.
```

Fig. 3. Simulation algorithm to obtain the average travel time of S/R mechanism.

(b) the height of each cell is 4.5 m, and the width is 4.5 m;
(c) the VP travels at 1 m/s and the HPs travel at 2 m/s.

Configuration 2:

(a) the number of total cells in the rack is 2592;
(b) the height of each cell is 1.5 m, and the width is 1.5 m;
(c) the same as (c) of Configuration 1.

The area of the rack used in Configuration 2 is the same as that of Configuration 1, but the dimensions of each cell are smaller, which yields more cells in the rack.

A sequence of 100 000 jobs is used in the simulation. The simulation results are obtained for two different cases, respectively. The first case gives the compared results under different b when α is fixed. The second one gives the comparison under different α when the *shape factor* b is fixed. Parts of the results are shown in Tables 3 and 4. The formula used to calculate the '% deviation' is

$$\%\text{Deviation} = \frac{\text{model results} - \text{simulation results}}{\text{simulation results}} \times 100\%.$$

Table 3 gives the simulation results with $\alpha = 0.5$.

From Table 3a, it can be observed that the maximum '% deviation' is less than 3%, and this shows that the continuous model performs quite well. Comparing the results from Tables 3a and

Table 3
Simulation results when $\alpha = 0.5$

No. of tiers	No. of bays	Cells in rack	Shape factor, b	Simulation results	Model results	% Deviation
(a) *Simulation result under Configuration* 1						
1	288	288	0.01	540.81	541.97	0.21
9	32	288	0.56	78.80	80.35	1.97
12	24	288	1.00	73.02	74.84	2.50
17	17	289	2.00	78.14	80.36	2.84
24	12	288	4.00	96.90	99.43	2.62
48	6	288	16.00	180.93	183.73	1.55
96	3	288	64.00	358.20	361.73	0.99
288	1	288	576.00	1074.08	1080.56	0.60
(b) *Simulation result under Configuration* 2						
3	864	2592	0.01	539.43	541.97	0.47
27	96	2592	0.56	79.69	80.35	0.82
36	72	2592	1.00	74.20	74.84	0.87
51	51	2601	2.00	79.56	80.36	1.00
72	36	2592	4.00	98.55	99.43	0.90
144	18	2592	16.00	182.62	183.73	0.61
288	9	2592	64.00	360.41	361.73	0.37
864	3	2592	576.00	1077.28	1080.56	0.30

Table 4
Simulation result when $b = 1$ (under Configuration 1)

α	Simulation results	Model results	% Deviation
0.1	80.03	81.59	1.96
0.2	76.78	78.61	2.39
0.3	74.39	76.52	2.87
0.4	72.97	75.28	3.16
0.5	73.02	74.84	2.49
0.6	73.21	75.16	2.67
0.7	74.23	76.21	2.67
0.8	75.95	77.93	2.61
0.9	78.06	80.29	2.86
1	80.93	83.25	2.86

Tier = 12, bay = 24, $b = 1$.

b, it can be seen that if the *shape factor b* is the same, then with the increased number of cells in the rack, the '% deviation' between the simulation and the continuous model becomes smaller; this accords with the fact that in the latter case the discrete rack is closer to a continuous pick face.

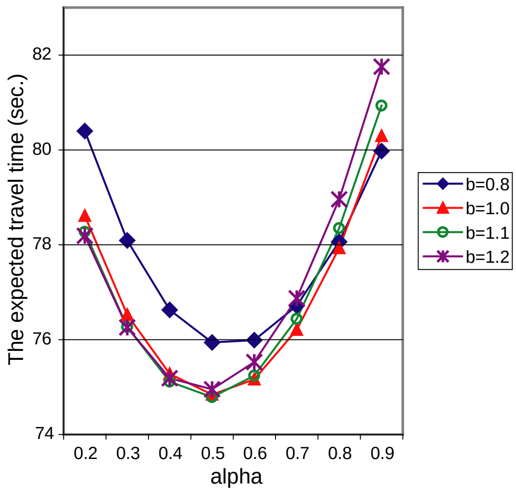

Fig. 4. The expected travel time vs. α.

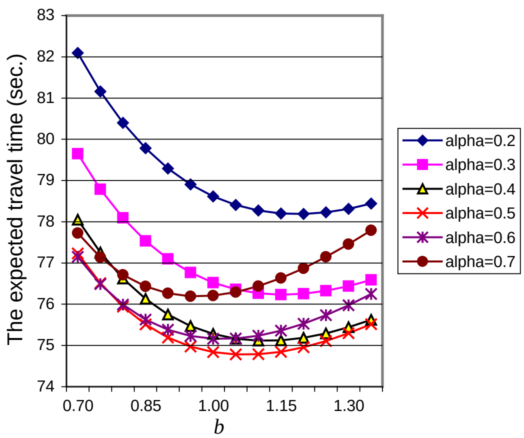

Fig. 5. The expected travel time vs. b.

From Table 4, it can be seen that with fixed *shape factor* b, the continuous model also gives satisfactory results in the cases of different values of α.

6. Sensitivity analysis on b and α

The influences of b and α on the expected travel time obtained from our models are investigated in this section.

In order to find the relationship between b, α and the expected travel time, a calculation based on Eq. (16) is conducted. In the calculation the range of b is limited to $[0.1, 5]$, and the interval

Table 5
Optimal α for given b

b	0–0.3	0.35–0.60	0.65–0.85	0.90–1.20	1.25–2.20	2.25–2.70	2.75–5.00
Optimal α	0.55	0.60	0.55	0.50	0.45	0.40	0.45

Table 6
Optimal b for given α

α	0–0.20	0.25–0.45	0.5–0.75	0.80–1.00
Optimal b	1.20–1.25	1.10–1.15	0.95–1.05	0.80–0.90

between the adjacent b is 0.05. At the same time, the value of α varies from 0 to 1 with equal interval of 0.05. Parts of the results are shown in Figs. 4 and 5.

From Figs. 4 and 5, it is easy to observe:

(1) Optima of α which yield the minimal expected travel time are around 0.5, and vary with the values of b as presented in Table 5.
(2) Optima of b that yield the minimal expected travel time lie in the range of 0.8–1.25, and vary with the values of α as presented in Table 6.

What can also be observed from the calculation is that the global optimum of the expected travel time is obtained around $\alpha = 0.5$ and $b = 1.05$.

7. Conclusions

We have presented a new kind of S/R mechanism that enables AS/RS to efficiently handle very heavy loads. The advantages of this SP-AS/RS include high throughput, high lifting capacity, and more flexible AS/RS rack configuration and high fault tolerance.

We have presented a continuous travel-time model for the new AS/RS under the *stay* dwell point policy. The model has been validated by the computer simulation results and the comparison shows that the model performs quite satisfactorily. The results of sensitivity analysis on b and α suggest that interleaving storage operations with retrieval operations in a batch of jobs can decrease the expected travel time, and an optimal *shape factor* b is found to be near 1. It is also observed that the global minimum of the expected travel time is obtained around $\alpha = 0.5$ and $b = 1.05$.

For future research, we suggest studying the policies for request sequencing (other than the FCFS used here), policies for storage assignment (other than the randomized policy here) and dwell point policies (e.g. return to middle). In fact, because this new AS/RS is entirely different from the conventional ones using stacker cranes, many existing results no longer apply and all the issues listed in Section 3 deserve further study. For instance, our preliminary research has shown that, by simply pre-fetching the loads using the multiple platforms, the average handling time for a batch of jobs can be greatly reduced.

In conclusion, the inherent concurrency of the SP-AS/RS has shown great potential for higher AS/RS performance.

Acknowledgements

We gratefully acknowledge the support by the Agency for Science, Technology and Research, Singapore, the Maritime and Port Authority of Singapore, Nanyang Technological University, Singapore, and the Singapore-MIT Alliance program, National University of Singapore. We would also like to give our thanks to the referees for their comments and suggestions.

Appendix A. Derivation of the travel time for the SP-AS/RS

As stated in Section 4.2, (x_1, y_1) and (x_2, y_2) denote the previous operation point and the target point of current job, respectively. (x_3, y_2) denotes the last operation point whose y coordinate is also y_2. For the case that the current job is storage; the previous operation and the last operation at tier y_2 are also storage operations, the movements of the VP and the HP on tier y_2 can be described as follows:

(1) VP takes a time of y_1 to travel from its dwell point to the I/O station and then spends a time of c_0 to receive the load from the I/O station. It then travels to tier y_2 in a time of y_2. At the same time, the corresponding HP of tier y_2 takes a time of x_3 units to reach bay 0. Hence, the time required for this portion is $\max(y_1 + c_0 + y_2, x_3)$.

(2) After spending a time of c to transfer the load from VP to HP, VP stays at tier y_2 according to the dwell point policy, whereas the corresponding HP takes x_2 to reach the storage cell. Then HP must use a time of c to put the load into the storage cell. After that, HP stays at that location. Thus, the time required for these movements is $c + x_2 + c$.

Thus the time for a storage operation under this case is

$$t_{sss} = \max(y_1 + c_0 + y_2, x_3) + x_2 + 2c. \tag{A.1}$$

Appendix B. Calculation of $E[\max(y_1 + y_2, x_3)]$

Let Z denote $\max(y_1 + y_2, x_3)$, we will give the expectation of Z. Because x_3 and $y_1 + y_2$ are independent of our analysis of the new mechanism we use, the probability distribution function of Z can be expressed as

$$F_Z(v) = P(Z \leqslant v) = P(x_3 \leqslant v)P(y_1 + y_2 \leqslant v). \tag{B.1}$$

Denote $M = y_1 + y_2$, the probability distribution function of M will be

$$F_M(v) = \int\int_{\substack{0 \leqslant u \leqslant b \\ 0 \leqslant t \leqslant b}} f_{y_1 y_2}(u, t)\,\mathrm{d}u\,\mathrm{d}t = \int\int_S f_{y_1}(u) f_{y_2}(t)\,\mathrm{d}u\,\mathrm{d}t = \int\int_S \frac{1}{b^2}\,\mathrm{d}u\,\mathrm{d}t,$$

where S denotes the intersection of the areas with $u + t \leqslant v$, $0 \leqslant u \leqslant b$ and $0 \leqslant t \leqslant b$.

Solving the above equation, we have

$$F_M(v) = \begin{cases} 0, & v \leqslant 0, \\ \dfrac{v^2}{2b^2}, & 0 \leqslant v \leqslant b, \\ \dfrac{2v}{b} - \dfrac{v^2}{2b^2} - 1, & b \leqslant v \leqslant 2b, \\ 1, & v \geqslant 2b. \end{cases} \tag{B.2}$$

Substituting Eqs. (B.2) and (2) into Eq. (B.1), we have

When $0 < b \leqslant \frac{1}{2}$

$$F_Z(v) = \begin{cases} 0, & v \leqslant 0, \\ \dfrac{v^3}{2b^2}, & 0 \leqslant v \leqslant b, \\ \dfrac{2v^2}{b} - \dfrac{v^3}{2b^2} - v, & b \leqslant v \leqslant 2b, \\ v, & 2b \leqslant v \leqslant 1, \\ 1, & v \geqslant 1 \end{cases} \quad \text{and} \quad f_Z(v) = \begin{cases} \dfrac{3v^2}{2b^2}, & 0 \leqslant v \leqslant b, \\ \dfrac{4v}{b} - \dfrac{3v^2}{2b^2} - 1, & b \leqslant v \leqslant 2b, \\ 1, & 2b \leqslant v \leqslant 1, \\ 0, & \text{otherwise.} \end{cases} \tag{B.3}$$

When $\frac{1}{2} \leqslant b \leqslant 1$

$$F_Z(v) = \begin{cases} 0, & v \leqslant 0, \\ \dfrac{v^3}{2b^2}, & 0 \leqslant v \leqslant b, \\ \dfrac{2v^2}{b} - \dfrac{v^3}{2b^2} - v, & b \leqslant v \leqslant 1, \\ \dfrac{2v}{b} - \dfrac{v^2}{2b^2} - 1, & 1 \leqslant v \leqslant 2b, \\ 1, & v \geqslant 2b \end{cases} \quad \text{and} \quad f_Z(v) = \begin{cases} \dfrac{3v^2}{2b^2}, & 0 \leqslant v \leqslant b, \\ \dfrac{4v}{b} - \dfrac{3v^2}{2b^2} - 1, & b \leqslant v \leqslant 1, \\ \dfrac{2}{b} - \dfrac{v}{b^2}, & 1 \leqslant v \leqslant 2b, \\ 0, & \text{otherwise.} \end{cases} \tag{B.4}$$

When $b \geqslant 1$

$$F_Z(v) = \begin{cases} 0, & v \leqslant 0, \\ \dfrac{v^3}{2b^2}, & 0 \leqslant v \leqslant 1, \\ \dfrac{v^2}{2b^2}, & 1 \leqslant v \leqslant b, \\ \dfrac{2v}{b} - \dfrac{v^2}{2b^2} - 1, & b \leqslant v \leqslant 2b, \\ 1, & v \geqslant 2b \end{cases} \quad \text{and} \quad f_Z(v) = \begin{cases} \dfrac{3v^2}{2b^2}, & 0 \leqslant v \leqslant 1, \\ \dfrac{v}{b^2}, & 1 \leqslant v \leqslant b, \\ \dfrac{2}{b} - \dfrac{v}{b^2}, & b \leqslant v \leqslant 2b, \\ 0, & \text{otherwise.} \end{cases} \tag{B.5}$$

Hence, based on Eqs. (B.3)–(B.5), the expected value of Z is expressed as

$$E[Z]=\int_v v f_Z(v)\,\mathrm{d}v=\begin{cases}\dfrac{7}{12}b^2+\dfrac{1}{2}, & 0<b\leqslant\dfrac{1}{2},\\ -\dfrac{1}{12}b^2+\dfrac{4}{3}b+\dfrac{1}{3b}-\dfrac{1}{24b^2}-\dfrac{1}{2}, & \dfrac{1}{2}\leqslant b\leqslant 1,\\ b+\dfrac{1}{24b^2}, & b\geqslant 1.\end{cases}$$

Appendix C. Calculation of $E[\max(y_2, x_3)]$

Let Z denote $\max(y_2,x_3)$, and because that the probability distribution functions of y_2 and x_3 are independent, we have

$$F_Z(v)=P(Z\leqslant v)=P(\max(y_2,x_3)\leqslant v)=P(y_2\leqslant v)P(x_3\leqslant v).$$

Incorporating (2), it has,

When $0<b\leqslant 1$

$$f_Z(v)=\begin{cases}\dfrac{2}{b}v, & 0\leqslant v\leqslant b,\\ 1, & b\leqslant v\leqslant 1,\\ 0, & \text{otherwise}\end{cases}\quad\text{for}\quad F_Z(v)=\begin{cases}\dfrac{1}{b}v^2 & 0\leqslant v\leqslant b,\\ v, & b\leqslant v\leqslant 1,\\ 1, & v\geqslant 1.\end{cases}\tag{C.1}$$

When $b\geqslant 1$

$$f_Z(v)=\begin{cases}\dfrac{2}{b}v, & 0\leqslant v\leqslant 1,\\ \dfrac{1}{b}, & 1\leqslant v\leqslant b,\\ 0, & \text{otherwise}\end{cases}\quad\text{for}\quad F_Z(v)=\begin{cases}\dfrac{1}{b}v^2, & 0\leqslant v\leqslant 1,\\ \dfrac{v}{b}, & 1\leqslant v\leqslant b,\\ 1, & v\geqslant b.\end{cases}\tag{C.2}$$

Therefore, based on Eqs. (C.1) and (C.2) we have

$$E[Z]=\int_v v f_Z(v)\,\mathrm{d}v=\begin{cases}\dfrac{1}{6}b^2+\dfrac{1}{2}, & 0<b\leqslant 1,\\ \dfrac{b}{2}+\dfrac{1}{6b}, & b\geqslant 1.\end{cases}$$

Appendix D. Calculation of $E[\max(|y_1-y_2|, |x_3-x_2|+x_2)]$

Let Z denote the expression $\max(|y_1-y_2|,|x_3-x_2|+x_2)$, we have

$$F_Z(v)=P(Z\leqslant v)=P(\max(|y_1-y_2|,|x_3-x_2|+x_2)\leqslant v).\tag{D.1}$$

Because $|y_1 - y_2|$ and $|x_3 - x_2| + x_2$ are independent, then the equation above can be expressed as

$$P(Z \leqslant v) = P(|y_1 - y_2| \leqslant v)P(|x_3 - x_2| + x_2 \leqslant v). \tag{D.2}$$

First, we consider $P(|y_1 - y_2| \leqslant v)$

$$\begin{aligned} P(|y_1 - y_2| \leqslant v) &= P(-v \leqslant y_1 - y_2 \leqslant v) = P(y_2 - v \leqslant y_1 \leqslant y_2 + v) \\ &= \int\int_{Q_v} f_{y_1 y_2}(u,t), \end{aligned}$$

where Q_v is the area bounded by the lines: $u = t + v$, $u = t - v$, $0 \leqslant u \leqslant b$ and $0 \leqslant t \leqslant b$.

Then, $P(|y_1 - y_2| \leqslant v) = \int_0^{b-v} \int_0^{t+v} f_{y_1 y_2}(u,t)\,\mathrm{d}u\,\mathrm{d}t + \int_{b-v}^{v} \int_0^{b} f_{y_1 y_2}(u,t)\,\mathrm{d}u\,\mathrm{d}t + \int_v^b \int_{t-v}^{b} f_{y_1 y_2}(u,t)\,\mathrm{d}u\,\mathrm{d}t$.

As y_1 and y_2 are independent and have the same probability distribution function, it is easy to get

$$f_{y_1 y_2}(u,t) = f_{y_1}(u) \times f_{y_2}(t) = \begin{cases} \dfrac{1}{b^2}, & 0 \leqslant u \leqslant b \text{ and } 0 \leqslant t \leqslant b, \\ 0, & \text{otherwise.} \end{cases}$$

So,

$$P(|y_1 - y_2| \leqslant v) = \begin{cases} \dfrac{2v}{b} - \dfrac{v^2}{b^2}, & 0 \leqslant v \leqslant b, \\ 1, & v \geqslant b. \end{cases} \tag{D.3}$$

Using the same method to calculate $P(|x_3 - x_2| + x_2 \leqslant v)$, we have

$$\begin{aligned} P(|x_3 - x_2| + x_2 \leqslant v) &= P(|x_3 - x_2| \leqslant v - x_2) = P(x_2 - v \leqslant x_3 - x_2 \leqslant v - x_2) \\ &= P(2x_2 - v \leqslant x_3 \leqslant v). \end{aligned}$$

Also consider the area bounded by the lines: $u = v$, $u = 2t - v$, $u = t - v$, $0 \leqslant u \leqslant 1$ and $0 \leqslant t \leqslant 1$.

We have

$$P(2x_2 - v \leqslant x_3 \leqslant v) = \begin{cases} \displaystyle\int_0^{v/2} \int_0^{v} f_{x_2 x_3}(t,u)\,\mathrm{d}u\,\mathrm{d}t + \int_{v/2}^{v} \int_{2t-v}^{v} f_{x_2 x_3}(t,u)\,\mathrm{d}u\,\mathrm{d}t, & 0 \leqslant v \leqslant 1, \\ \displaystyle\int_0^{v/2} \int_0^{1} f_{x_2 x_3}(t,u)\,\mathrm{d}u\,\mathrm{d}t + \int_{v/2}^{1} \int_{2t-v}^{1} f_{x_2 x_3}(t,u)\,\mathrm{d}u\,\mathrm{d}t, & 1 \leqslant v \leqslant 2, \\ 1, & v \geqslant 2. \end{cases}$$

Consider the probability density function of x_2 and x_3, and they are independent, so

$$f_{x_2x_3}(t,u) = f_{x_2}(t) \times f_{x_3}(u) = \begin{cases} 1, & 0 \leqslant u \leqslant 1 \text{ and } 0 \leqslant t \leqslant 1, \\ 0, & \text{otherwise.} \end{cases}$$

Now we can see

$$P(|x_3 - x_2| + x_2 \leqslant v) = P(2x_2 - v \leqslant x_3 \leqslant v) = \begin{cases} \frac{3}{4}v^2, & 0 \leqslant v \leqslant 1, \\ v - \frac{1}{4}v^2, & 1 \leqslant v \leqslant 2, \\ 1, & v \geqslant 2. \end{cases} \tag{D.4}$$

Substituting Eqs. (D.2)–(D.4) into Eq. (D.1) yields,
when $0 < b \leqslant 1$

$$F_Z(v) = \begin{cases} \frac{3}{4}v^2\left(\frac{2v}{b} - \frac{v^2}{b^2}\right), & 0 \leqslant v \leqslant b, \\ \frac{3}{4}v^2, & b \leqslant v \leqslant 1, \\ v - \frac{1}{4}v^2, & 1 \leqslant v \leqslant 2, \\ 1, & v \geqslant 2 \end{cases} \quad \text{and} \quad f_Z(v) = \begin{cases} \frac{9v^2}{2b} - \frac{3v^3}{b^2}, & 0 \leqslant v \leqslant b, \\ \frac{3}{2}v, & b \leqslant v \leqslant 1, \\ 1 - \frac{1}{2}v, & 1 \leqslant v \leqslant 2, \\ 0, & \text{otherwise.} \end{cases} \tag{D.5}$$

When $1 < b \leqslant 2$

$$F_Z(v) = \begin{cases} \frac{3}{4}v^2\left(\frac{2v}{b} - \frac{v^2}{b^2}\right), & 0 \leqslant v \leqslant 1, \\ \left(\frac{2v}{b} - \frac{v^2}{b^2}\right)\left(v - \frac{1}{4}v^2\right), & 1 \leqslant v \leqslant b, \\ v - \frac{1}{4}v^2, & b \leqslant v \leqslant 2, \\ 1, & v \geqslant 2 \end{cases} \quad \text{and}$$

$$f_Z(v) = \begin{cases} \frac{9v^2}{2b} - \frac{3v^3}{b^2}, & 0 \leqslant v \leqslant 1, \\ \frac{4v}{b} - \frac{3v^2}{2b} - \frac{3v^2}{b^2} + \frac{v^3}{b^2}, & 1 \leqslant v \leqslant b, \\ 1 - \frac{1}{2}v, & b \leqslant v \leqslant 2, \\ 0, & \text{otherwise.} \end{cases} \tag{D.6}$$

When $b \geqslant 2$,

$$F_Z(v) = \begin{cases} \frac{3}{4}v^2\left(\frac{2v}{b} - \frac{v^2}{b^2}\right), & 0 \leqslant v \leqslant 1, \\ \left(\frac{2v}{b} - \frac{v^2}{b^2}\right)\left(v - \frac{1}{4}v^2\right), & 1 \leqslant v \leqslant 2, \\ \frac{2v}{b} - \frac{v^2}{b^2}, & 2 \leqslant v \leqslant b, \\ 1, & v \geqslant b \end{cases} \quad \text{and}$$

$$f_Z(v) = \begin{cases} \frac{9v^2}{2b} - \frac{3v^3}{b^2}, & 0 \leqslant v \leqslant 1, \\ \frac{4v}{b} - \frac{3v^2}{2b} - \frac{3v^2}{b^2} + \frac{v^3}{b^2}, & 1 \leqslant v \leqslant 2, \\ \frac{2}{b} - \frac{2v}{b^2}, & 2 \leqslant v \leqslant b, \\ 0, & \text{otherwise.} \end{cases} \tag{D.7}$$

Based on Eqs. (D.5)–(D.7) we have

$$E[Z] = \int_v v f_Z(v)\,\mathrm{d}v = \begin{cases} \frac{1}{40}b^3 + \frac{5}{6}, & 0 < b \leqslant 1, \\ -\frac{b^3}{120} + \frac{b^2}{12} + \frac{1}{6b} - \frac{1}{20b^2} + \frac{2}{3}, & 1 \leqslant b \leqslant 2, \\ \frac{b}{3} + \frac{5}{6b} - \frac{19}{60b^2}, & b \geqslant 2. \end{cases}$$

Appendix E. Calculation of $E[\max(|y_1 - y_2|, 2x_2)]$

Let Z denote $\max(|y_1 - y_2|, 2x_2)$, we have $F_Z(v) = P(\max(|y_1 - y_2|, 2x_2) \leqslant v) = P(|y_1 - y_2| \leqslant v)$ $P(2x_2 \leqslant v)$

Recall that

$$P(|y_1 - y_2| \leqslant v) = \begin{cases} 0, & v \leqslant 0, \\ \frac{2v}{b} - \frac{v^2}{b^2}, & 0 \leqslant v \leqslant b, \\ 1, & v \geqslant b \end{cases} \quad \text{and} \quad P(2x_2 \leqslant v) = \begin{cases} 0, & v \leqslant 0, \\ \frac{v}{2}, & 0 \leqslant v \leqslant 2, \\ 1, & v \geqslant 2. \end{cases}$$

We have

When $0 < b \leqslant 2$

$$F_Z(v) = \begin{cases} 0, & v \leqslant 0, \\ \dfrac{v^2}{b} - \dfrac{v^3}{2b^2}, & 0 \leqslant v \leqslant b, \\ \dfrac{v}{2}, & b \leqslant v \leqslant 2, \\ 1, & v \geqslant 2 \end{cases} \quad \text{and} \quad f_Z(v) = \begin{cases} \dfrac{2v}{b} - \dfrac{3v^2}{2b^2}, & 0 \leqslant v \leqslant b, \\ \dfrac{1}{2}, & b \leqslant v \leqslant 2, \\ 0, & \text{otherwise.} \end{cases} \tag{E.1}$$

When $b \geqslant 2$

$$F_Z(v) = \begin{cases} 0, & v \leqslant 0, \\ \dfrac{v}{2}\left(\dfrac{2v}{b} - \dfrac{v^2}{b^2}\right), & 0 \leqslant v \leqslant 2, \\ \dfrac{2v}{b} - \dfrac{v^2}{b^2}, & 2 \leqslant v \leqslant b, \\ 1, & v \geqslant b \end{cases} \quad \text{and} \quad f_Z(v) = \begin{cases} \dfrac{2v}{b} - \dfrac{3v^2}{2b^2}, & 0 \leqslant v \leqslant 2, \\ \dfrac{2}{b} - \dfrac{2v}{b^2}, & 2 \leqslant v \leqslant b, \\ 0, & \text{otherwise.} \end{cases} \tag{E.2}$$

Based on Eqs. (E.1) and (E.2), the expected value of Z can be obtained as

$$E[Z] = \int_v v f_Z(v) = \begin{cases} \dfrac{b^2}{24} + 1, & 0 < b \leqslant 2, \\ \dfrac{b}{3} + \dfrac{4}{3b} - \dfrac{2}{3b^2}, & b \geqslant 2. \end{cases}$$

Appendix F. Calculation of $E[\max(2x_2, y_2)]$

Let Z denote $\max(2x_2, y_2)$. Note that x_2 and y_2 are independent, then the probability distribution function of Z can be expressed as

$$F_Z(v) = P(Z \leqslant v) = P(2x_2 \leqslant v)P(y_2 \leqslant v).$$

Because

$$P(2x_2 \leqslant v) = \begin{cases} 0, & v \leqslant 0, \\ \dfrac{v}{2}, & 0 \leqslant v \leqslant 2, \\ 1, & v \geqslant 2, \end{cases}$$

it has.

When $0 < b \leqslant 2$

$$F_Z(v) = \begin{cases} 0, & v \leqslant 0, \\ \dfrac{v^2}{2b}, & 0 \leqslant v \leqslant b, \\ \dfrac{v}{2}, & b \leqslant v \leqslant 2, \\ 1, & v \geqslant 2 \end{cases} \quad \text{and} \quad f_Z(v) = \begin{cases} \dfrac{v}{b}, & 0 \leqslant v \leqslant b, \\ \dfrac{1}{2}, & b \leqslant v \leqslant 2, \\ 0, & \text{otherwise.} \end{cases} \tag{F.1}$$

When $b \geqslant 2$

$$F_Z(v) = \begin{cases} 0, & v \leqslant 0, \\ \dfrac{v^2}{2b}, & 0 \leqslant v \leqslant 2, \\ \dfrac{v}{b}, & 2 \leqslant v \leqslant b, \\ 1, & v \geqslant b \end{cases} \quad \text{and} \quad f_Z(v) = \begin{cases} \dfrac{v}{b}, & 0 \leqslant v \leqslant 2, \\ \dfrac{1}{b}, & 2 \leqslant v \leqslant b, \\ 0, & \text{otherwise.} \end{cases} \tag{F.2}$$

Consequently, the expected value of Z becomes

$$E[Z] = \int_v v f_Z(v)\,\mathrm{d}v = \begin{cases} \dfrac{b^2}{12} + 1, & 0 < b \leqslant 2, \\ \dfrac{b}{2} + \dfrac{2}{3b}, & b \geqslant 2. \end{cases}$$

Appendix G. Calculation of $E[\max(y_2, |x_2 - x_3| + x_2)]$

Let Z denote $\max(y_2, |x_2 - x_3| + x_2)$, then

$$F_Z(v) = P(Z \leqslant v) = P(\max(y_2, |x_2 - x_3| + x_2)) = P(y_2 \leqslant v)P(|x_2 - x_3| + x_2 \leqslant v).$$

From the same calculation as given in Appendix D, we have

$$P(y_2 \leqslant v) = \begin{cases} 0, & v \leqslant 0, \\ \dfrac{v}{b}, & 0 < v \leqslant b, \\ 1, & v \geqslant b \end{cases} \quad \text{and} \quad P(|x_2 - x_3| + x_2 \leqslant v) = \begin{cases} \dfrac{3}{4}v^2, & 0 \leqslant v \leqslant 1, \\ v - \dfrac{1}{4}v^2, & 1 \leqslant v \leqslant 2, \\ 1, & v \geqslant 2. \end{cases}$$

Thus, when $0 < b \leqslant 1$

$$F_Z(v) = \begin{cases} \dfrac{3}{4b}v^3, & 0 \leqslant v \leqslant b, \\ \dfrac{3}{4}v^2, & b \leqslant v \leqslant 1, \\ v - \dfrac{1}{4}v^2, & 1 \leqslant v \leqslant 2, \\ 1, & v \geqslant 2 \end{cases} \quad \text{and} \quad f_Z(v) = \begin{cases} \dfrac{9}{4b}v^2, & 0 \leqslant v \leqslant b, \\ \dfrac{3}{2}v, & b \leqslant v \leqslant 1, \\ 1 - \dfrac{1}{2}v, & 1 \leqslant v \leqslant 2, \\ 0, & \text{otherwise.} \end{cases} \tag{G.1}$$

When $1 \leqslant b \leqslant 2$

$$F_Z(v) = \begin{cases} \frac{3}{4b}v^3, & 0 \leqslant v \leqslant 1, \\ \frac{v}{b}\left(v - \frac{v^2}{4}\right), & 1 \leqslant v \leqslant b, \\ v - \frac{1}{4}v^2, & b \leqslant v \leqslant 2, \\ 1, & v \geqslant 2 \end{cases} \quad \text{and} \quad f_Z(v) = \begin{cases} \frac{9}{4b}v^2, & 0 \leqslant v \leqslant 1, \\ \frac{2v}{b} - \frac{3v^2}{4b}, & 1 \leqslant v \leqslant b, \\ 1 - \frac{1}{2}v, & b \leqslant v \leqslant 2, \\ 0, & \text{otherwise.} \end{cases} \tag{G.2}$$

When $b \geqslant 2$

$$F_Z(v) = \begin{cases} \frac{3}{4b}v^3, & 0 \leqslant v \leqslant 1, \\ \frac{v}{b}\left(v - \frac{v^2}{4}\right), & 1 \leqslant v \leqslant 2, \\ \frac{v}{b}, & 2 \leqslant v \leqslant b, \\ 1, & b \leqslant v \end{cases} \quad \text{and} \quad f_Z(v) = \begin{cases} \frac{9}{4b}v^2, & 0 \leqslant v \leqslant 1, \\ \frac{2v}{b} - \frac{3v^2}{4b}, & 1 \leqslant v \leqslant 2, \\ \frac{1}{b}, & 2 \leqslant v \leqslant b, \\ 0, & \text{otherwise.} \end{cases} \tag{G.3}$$

Hence,

$$E[Z] = \int_v v f_Z(v)\,\mathrm{d}v = \begin{cases} \frac{1}{16}b^3 + \frac{5}{6}, & 0 < b \leqslant 1, \\ -\frac{1}{48}b^3 + \frac{1}{6}b^2 + \frac{1}{12b} + \frac{2}{3}, & 1 \leqslant b \leqslant 2, \\ \frac{b}{2} + \frac{5}{12b}, & b \geqslant 2. \end{cases}$$

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